Province of the
EASTERN CAPE
EDUCATION

LEARNER'S NAME:/ *LEERDER SE NAAM*:	Solns

GRADE 12/*GRAAD* 12	

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIORSERTIFIKAAT*

GRADE 12/*GRAAD 12*

SEPTEMBER 2024

MATHE4

MATHEMATICS P2/*WISKUNDE V2* SPECIAL ANSWER BOOK/*SPESIALE ANTWOORDEBOEK*

Marker/*Merker*				Moderator's Initials/*Moderator se paraaf*															
Question/ *Vraag*	Mark/ *Punt*			Initial/ Parafeer	Marks/ *Punte*			S M	Marks/ *Punte*			D M	Marks/ *Punte*			P M	Marks/ *Punte*			NM
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8																				
9																				
10																				
TOTAL/ *TOTAAL*																				

This special answer book consists of 21 pages. /
Hierdie spesiale antwoordeboek bestaan uit 21 bladsye.

QUESTION 1/*VRAAG* 1

41 50 55 | 63 64 66 71 78 80 82 84 88 88 96 98

57,3 82 64 55 50 41

71 78 88 98 96

63 66 80 84 88

15

	Solution/*Oplossing*	Marks/ *Punte*
1.1.1	Mode = 88 ✓	(1) 1
1.1.2	Range = 98 − 41 = 57 ✓	(1) 1
1.1.3	$\bar{x} = \frac{41 + 50 + \cdots + 98}{15}$ $= \frac{1104}{15}$ ✓ $= 73,6$ ✓ ans only 2/2	(2) 2
1.1.4	$\sigma = 16,3$ ✓	(1) 1
1.1.5	$\bar{x} - 1\sigma = 73,6 - 16,3 = 57,3$ ✓ $< 57,3$ $\therefore$ 3 drivers ✓	(3) 3
1.2	$\bar{x}_s = 75$ kg $\quad \bar{x}_n = 75$ kg $n = \frac{1000}{75}$ ✓ $= 13,33\ldots$ ✓ $\therefore$ $13,33\ldots - 8$ ✓ $= 5,33\ldots$ $\therefore$ 5 ✓ more people	(4) 4
		[12]

QUESTION 2/*VRAAG* 2

TEST / TOETS A (x)	39	33	35	44	37	40	24	31	30	5
TEST / TOETS B (y)	41	45	48	40	47	42	37	44	43	24

	Solution/*Oplossing*	**Marks/ *Punte***
2.1	outlier (5; 24) ✓	(1)
2.2	$A = 25,48$ ✓ $B = 0,49$ ✓ $\therefore y = 25,48 + 0,49x$ ✓	(3)
2.3	$y = 25,48 + 0,49(14)$ ✓ $= 32,34$ $\therefore$ 32 ✓ $\in \mathbb{N}_0$	(2)
2.4	$r = 0,79$ ✓ substantial ✓ positive correlation.	(2)
		[8]

1

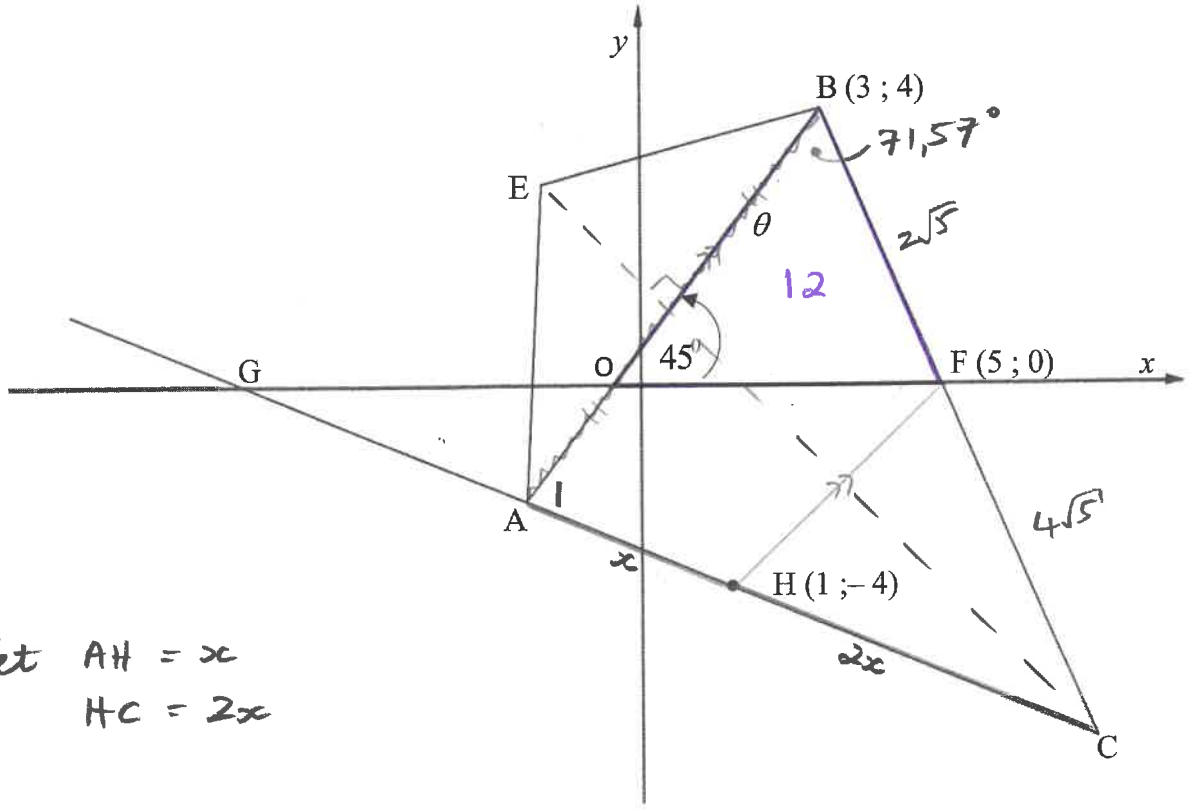
3

2

2

- accept "strong" this time
- finality to be sent to school for Finals.

QUESTION 3/*VRAAG* 3



let AH = x
HC = 2x

	Solution/*Oplossing*	Marks/ *Punte*
3.1	$BF = \sqrt{(4-0)^2 + (3-5)^2}$ ✓ $= \sqrt{20}$ $= \sqrt{4.5}$ $= 2\sqrt{5}$ ✓ ssf ⟶	(2)
3.2	$m_{BF} = \frac{4-0}{3-5}$ ✓ $= -2$ ✓ ⟶	(2)

2

2

3.3

$\tan B\hat{F}x = -2$ ✓

$Nf\hat{} = 63,43...°$

$\tan^{-1}$

II : $B\hat{F}x = 116,56...$ ✓

$116,56...° = \theta + 45°$ Ext $\angle$ Δ

$71,57° = \theta$ ✓

3

(3)

3.4

$m_{HF} = \frac{0-(-4)}{5-1}$ ✓ $= 1$ ✓

$m_{AB} = \tan 45° = 1$ ✓

$\therefore$ $HF \parallel AB$ ✓R $m_{AB} = m_F = 1$

4

(4)

3.5

kite (convex) ✓

1

(1)

3.6

$\frac{CB}{BF} = \frac{CA}{AH}$ ✓S

$\frac{CB}{2\sqrt{5}} = \frac{3x}{x}$ ✓

$CB = 6\sqrt{5}$ ✓

$\therefore AC = 6\sqrt{5}$

$= 13,42$ ✓S

line $\parallel$ 1 side of Δ must be used

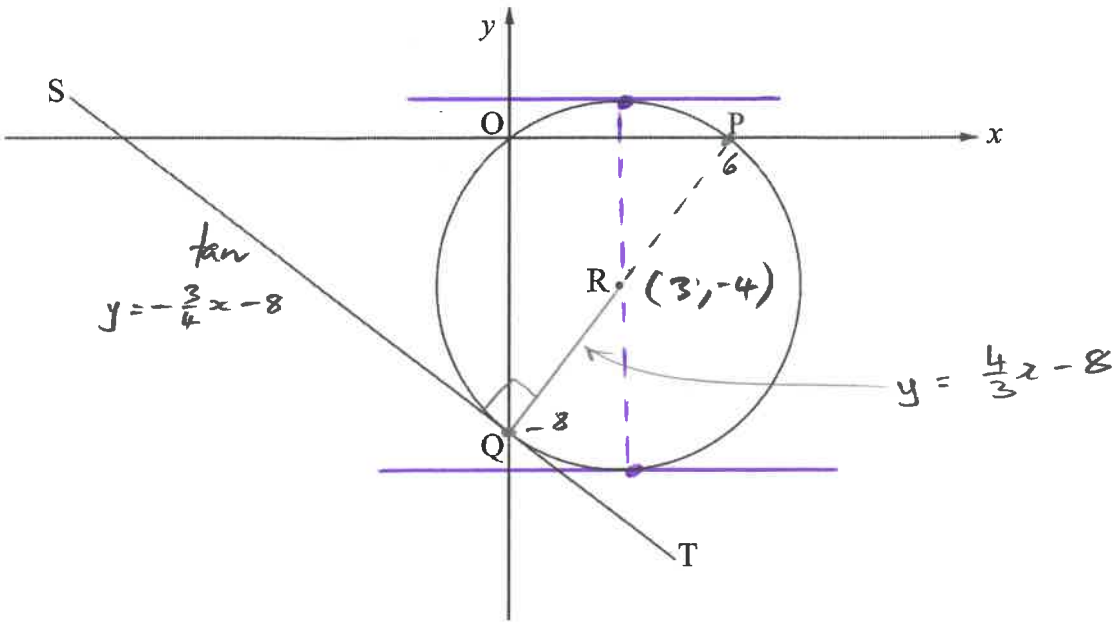
adj sides kite = $6\sqrt{5}$ $3\sqrt{20}$

4

(4)

3.7	$\hat{A}_1 = 71,57°$ ^'s opp = sides	
	$H\hat{C}B = 36,86°$ ✓S sum ^'s in $\Delta = 180°$	
	$\therefore$ area ΔABC	
	$= \frac{1}{2}(13,42)(13,42)\sin 36,86°$ ✓	3
	$= 54,01\ldots$	
	$\therefore$ area AOFC	
	$= 54,01\ldots - 12$	
	$= 42,02\ u^2$ ✓ 42	(3)
		[19]

QUESTION 4/*VRAAG 4*



	Solution/*Oplossing*	Marks/ *Punte*
4.1.1	y_{int} $y = -8$ $Q(0;-8)$ ✓ ✓	2 (2)
4.1.2	$m_{QR} = \frac{4}{3}$ ✓S tan ⊥ rad $y = \frac{4}{3}x + c$ $c = -8$ ✓ $y_Q = -8$ $\therefore y = \frac{4}{3}x - 8$ ✓	3 (3)

4.1.3

x int: $0 = \frac{4}{3}x - 8$

$8 = \frac{4}{3}x$

$6 = x$

$\therefore P(6;0)$ ✓✓ 2

(2)

4.1.4

$x_R = \frac{6+0}{2}$ $y_R = \frac{0+-8}{2}$ P(6;0) Q(0;-8)

$= 3$ ✓method $= -4$ radii

$\therefore R(3;-4)$ ✓✓ 3

(3)

4.1.5

$r^2 = (-8-(-4))^2 + (0-3)^2 = 25$ ✓

$\therefore (x-3)^2 + (y+4)^2 = 25$ ✓✓ 3

(3)

4.1.6

$k = -4+5$ or $-4-5$ ✓method $y_R \pm r$

$= 1$ ✓ $= -9$ ✓ 3

(3)

4.2

$x^2 - 2\sin\theta.x + (-\sin\theta)^2 + y^2 - 4\sin\theta\, y + (-2\sin\theta)^2 = -2 + \sin^2\theta + 4\sin^2\theta$ ✓cts

$(x-\sin\theta)^2 + (y-2\sin\theta)^2 = -2 + 5\sin^2\theta$ ✓RHS

$r = \sqrt{-2+5\sin^2\theta}$

$= \sqrt{-2+5.(1)^2}$ ✓ max $\sin\theta = 1$ ✓

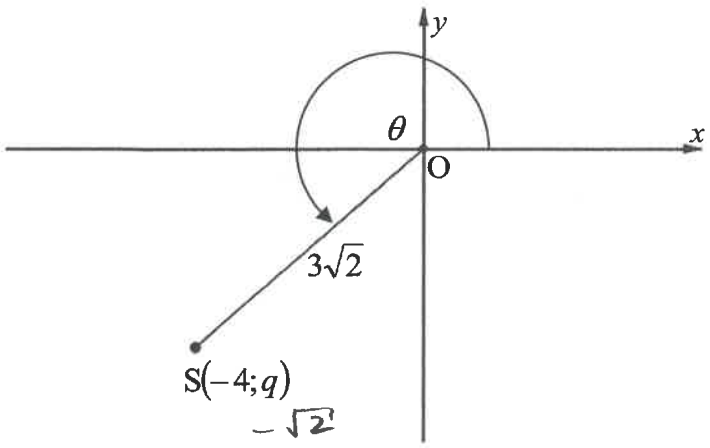
$= \sqrt{3}$

$= 1,73$ ✓ $\sqrt{3}$ 5

(5)

[21]

QUESTION 5/*VRAAG 5*



	Solution/*Oplossing*	Marks/ *Punte*
5.1.1	$(-4)^2 + q^2 = (3\sqrt{2})^2$ ✓ Pythag $q^2 = 2$ 9.2 18−16 $q = \pm\sqrt{2}$ reject + $q = -\sqrt{2}$ ✓	(2)
5.1.2	$\sin(\theta + 45°) = \sin\theta\cos 45° + \cos\theta \sin 45°$ ✓ $= \left(\frac{-\sqrt{2}}{3\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{-4}{3\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$ θ ✓ 45° ✓ $= -\frac{\sqrt{2}}{6} - \frac{4}{6}$ OR $-\frac{1}{3\sqrt{2}} - \frac{2}{3}$ $= \frac{-\sqrt{2}-4}{6}$ ✓ 	(4)
5.1.3	$\cos(2\theta - 360°)$ + 360° $= \cos 2\theta$ ✓ $= 2\cos^2\theta - 1$ ✓ OR $1 - 2\sin^2\theta$ OR $\cos^2\theta - \sin^2\theta$ $= 2\left(\frac{-4}{3\sqrt{2}}\right)^2 - 1$ ✓ $= 2\left(\frac{16}{9.2}\right) - 1$ $2.\frac{16}{18}$ $\frac{16}{9}$ $= \frac{16}{9} - 1$ 16−9 = 7 $= \frac{7}{9}$ ✓	(4)

2

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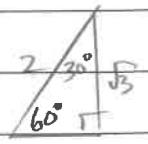
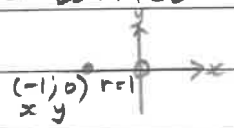
5.2

- $\sin(90^\circ - \theta) = \cos\theta$
- $\cos 480^\circ = \cos 120^\circ$
  $= \cos(180^\circ - 60^\circ)$
  $= -\cos 60^\circ$
  $= -\frac{1}{2}$



- $\sin 150^\circ = \sin(90^\circ + 60^\circ)$
  $= \cos 60^\circ$
  $= \frac{1}{2}$
- $\tan 180^\circ = \frac{0}{-1}$
  $= 0$



- $\cos(180^\circ - \theta) = -\cos\theta$

$$\frac{(\cos\theta)(-\frac{1}{2}) + (-\cos\theta)}{(\cos\theta)(\frac{1}{2}) - (0)}$$

$$= \frac{-\frac{1}{2}\cos\theta - \cos\theta}{\frac{1}{2}\cos\theta}$$

$$= \frac{-\frac{3}{2}\cos\theta}{\frac{1}{2}\cos\theta}$$

$$= -3$$

5

(5)

5.3

$$\frac{\cos x}{2\sin x \cos x} - \frac{\cos 2x}{2\sin x}$$

$$= \frac{1}{2\sin x} - \frac{\cos 2x}{2\sin x}$$

$$= \frac{1 - \cos 2x}{2\sin x}$$

$$= \frac{1 - (1 - 2\sin^2 x)}{2\sin x}$$

$$= \frac{1 - 1 + 2\sin^2 x}{2\sin x}$$

$$= \frac{2\sin^2 x}{2\sin x}$$

$$= \sin x$$

num

den

5

(5)

5.4.1

$\frac{\cos 60^\circ}{\sin x} - \frac{\sin 60^\circ}{\cos x} = 2$

$\frac{\cos 60^\circ \cos x - \sin 60^\circ \sin x}{\sin x \cos x} = 2$

$\cos(60^\circ + x) = 2 \sin x \cos x$

$\cos(x + 60^\circ) = \sin 2x$

$\cos(x + 60^\circ) = \cos(90^\circ - 2x)$

(3)

5.4.2

$\cos A = \cos B \qquad A = x + 60^\circ \qquad B = 90^\circ - 2x$

I $\cos B$ $\cos(360^\circ - B)$ IV

$\cos A = \cos B$	$\cos A = \cos(360^\circ - B)$
$x + 60^\circ = 90^\circ - 2x + k\,360^\circ$	$x + 60^\circ = 360^\circ - (90^\circ - 2x) + k\,360^\circ$
$3x = 30^\circ + k\,360^\circ$	$= 360^\circ - 90^\circ + 2x + k\,360^\circ$
$x = 10^\circ + k.120^\circ ; k \in \mathbb{Z}$	$-x = 210^\circ + k.360^\circ$
	$x = -210^\circ - k\,360^\circ ; k \in \mathbb{Z}$

(4)

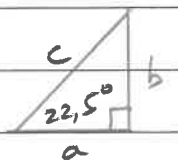
5.5

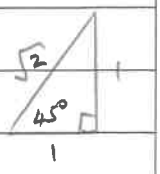
$\sin 2x = 2 \sin x \cos x \qquad x = 45^\circ$

$\sin 45^\circ = 2 \sin 22{,}5^\circ \cos 22{,}5^\circ$

$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 2\left(\frac{b}{c}\right)\left(\frac{a}{c}\right)$

$\frac{\sqrt{2}}{2} = \frac{2ab}{c^2}$

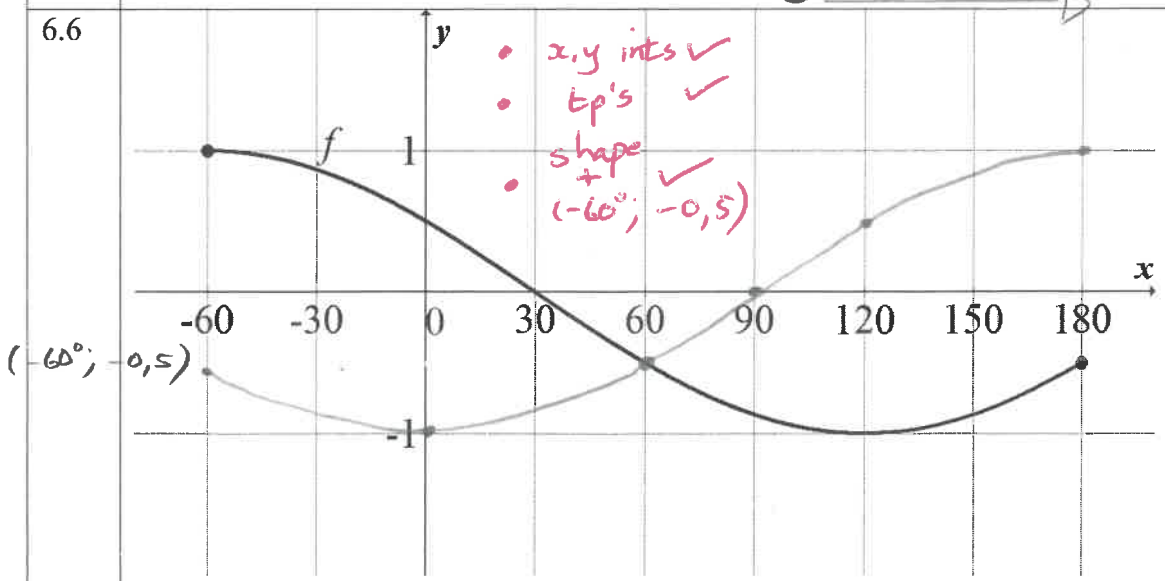




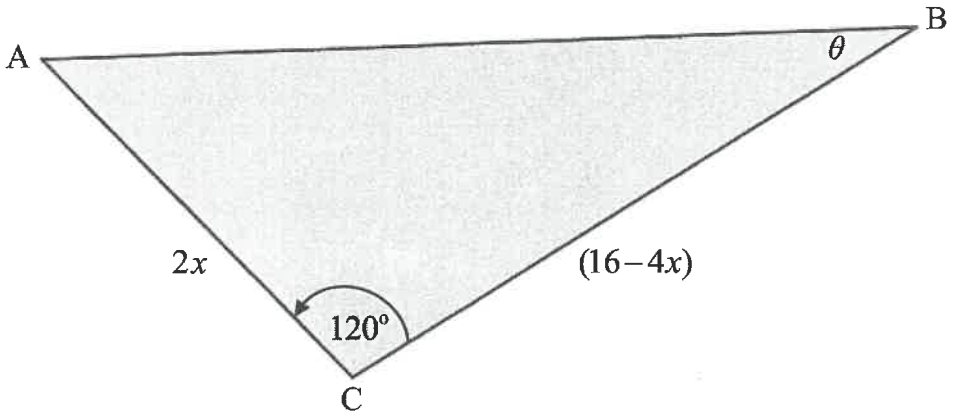
(5)

[32]

QUESTION 6/*VRAAG* 6

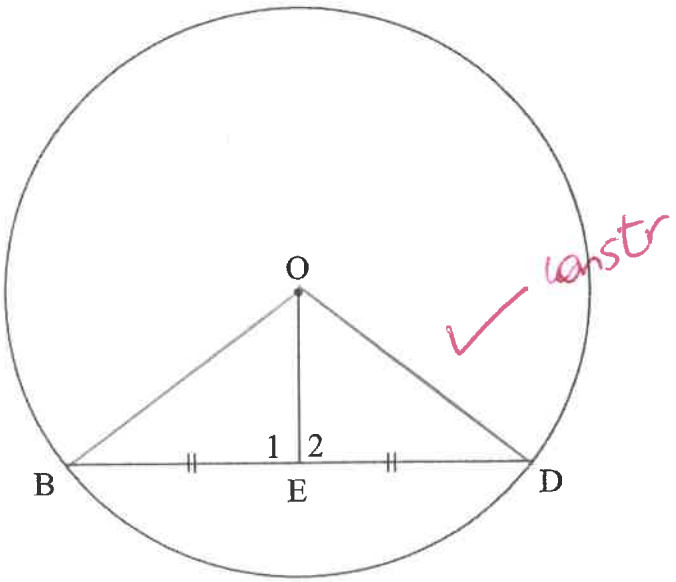
	Solution/*Oplossing*	Marks/ *Punte*
6.1	$360°$ ✓ → $\frac{360°}{1}$ ← "b" = 1	(1) 1
6.2	-1 ✓ →	(1) 1
6.3	f: $y \in [-1; 1]$ ✓ int ✓ not^n $f+1$: $y \in [0; 2]$ →	(2) 2
6.4	Incr $x \in (120°; 180°]$ → ✓ int ✓ not^n	(2) 2
6.5	• $60°$ → $y = \sin(x - 60° - 30°)$ ‖ $y = -(-\cos x)$ $= -\sin(x - 90°)$ ✓ ‖ $= \cos x$ ✓ $= -\sin(x + 270°)$ ‖ • refl x $-y = \cos x$ $= -\sin(270° + x)$ ‖ $y = -\cos x$ ✓ →	(3) 3
6.6		(3) 3
		[12]

QUESTION 7/*VRAAG 7*



	Solution/*Oplossing*	**Marks/ *Punte***
7.1	area $\Delta ABC = \frac{1}{2}(2x)(16-4x)\sin 120°$ ✓ $\sin 120° = \sin(180° - 60°)$ $= \sin 60°$ ✓ $= \frac{\sqrt{3}}{2}$ $\therefore$ area $\Delta ABC = \frac{1}{2}(2x)(16-4x)\left(\frac{\sqrt{3}}{2}\right)$ $= x(8\sqrt{3} - 2\sqrt{3}x)$ $= 8\sqrt{3}x - 2\sqrt{3}x^2$ ✓	3 (3)
7.2	$x = \frac{-(8\sqrt{3})}{2(-2\sqrt{3})}$ ✓ ✓ $-\frac{b}{2a}$ $= 2$ ✓ GR $A' = 0$ $8\sqrt{3} - 4\sqrt{3}x = 0$ ✓ $x = 2$	3 (3)
		[6]

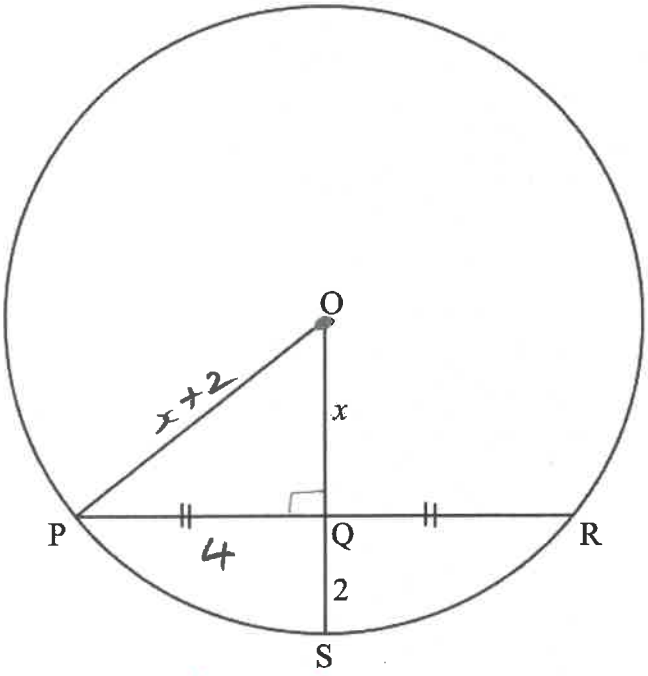
QUESTION 8/*VRAAG 8*



constr ✓

	Solution/*Oplossing*	Marks/ *Punte*
8.1	In Δ's OBE, ODE	
	1. OB = OD — radii ✓SR	
	2. OE = OE — common ✓S	
	3. BE = ED — given	
	$\therefore \Delta OBE \equiv \Delta ODE$ — SSS ✓R	
	$\therefore \hat{E}_1 = \hat{E}_2$ — $\Delta OBE \equiv \Delta ODE$ ✓SR	
	$\therefore \hat{E}_1 = \hat{E}_2 = 90°$ — $\angle$'s on str. line $= 180°$	
	$\therefore$ OE $\perp$ BD ▷	
		(5)

5



8.2.1	$OQP = 90°$ ✓S ✓R line from centre O to midpt of chord	(2)
8.2.2	$PQ = 4$ ✓ given bisected $OP = x + 2$ radii $(x+2)^2 = x^2 + 4^2$ ✓S Pythag $x^2 + 4x + 4 = x^2 + 16$ ✓ $4x = 12$ $x = 3$ ✓ $\therefore PO = 3 + 2$ $x+2$ $= 5$ ✓	(5)
		[12]

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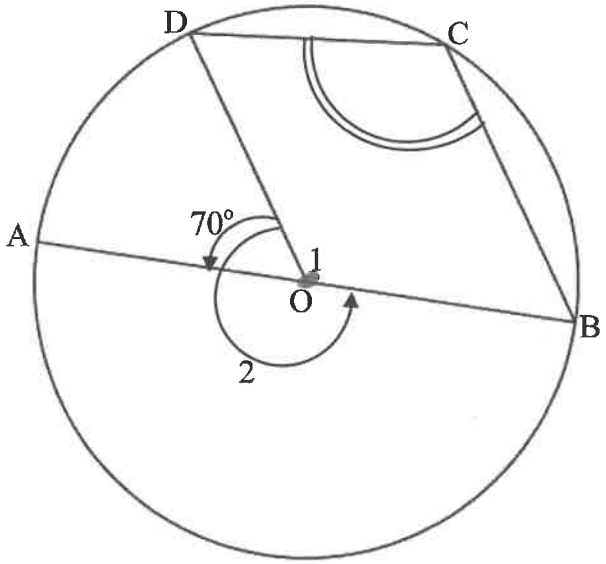
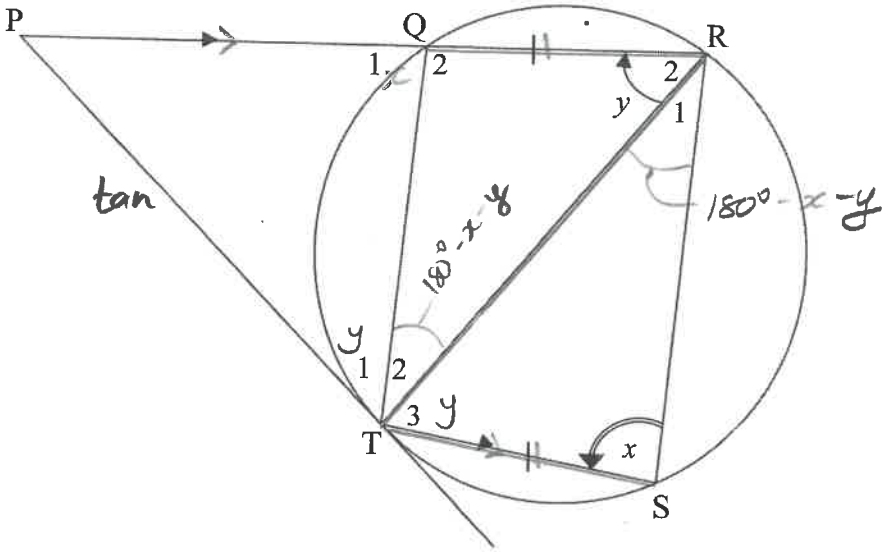
QUESTION 9/*VRAAG 9*



	Solution/*Oplossing*	**Marks/ *Punte***
9.1	$\hat{O}_2 = 250°$ ✓✓ ^'s on str line $= 180°$ $\hat{C} = 125°$ ✓✓ ^@centre $= 2 \times$ ^@ circum	5
		(5)



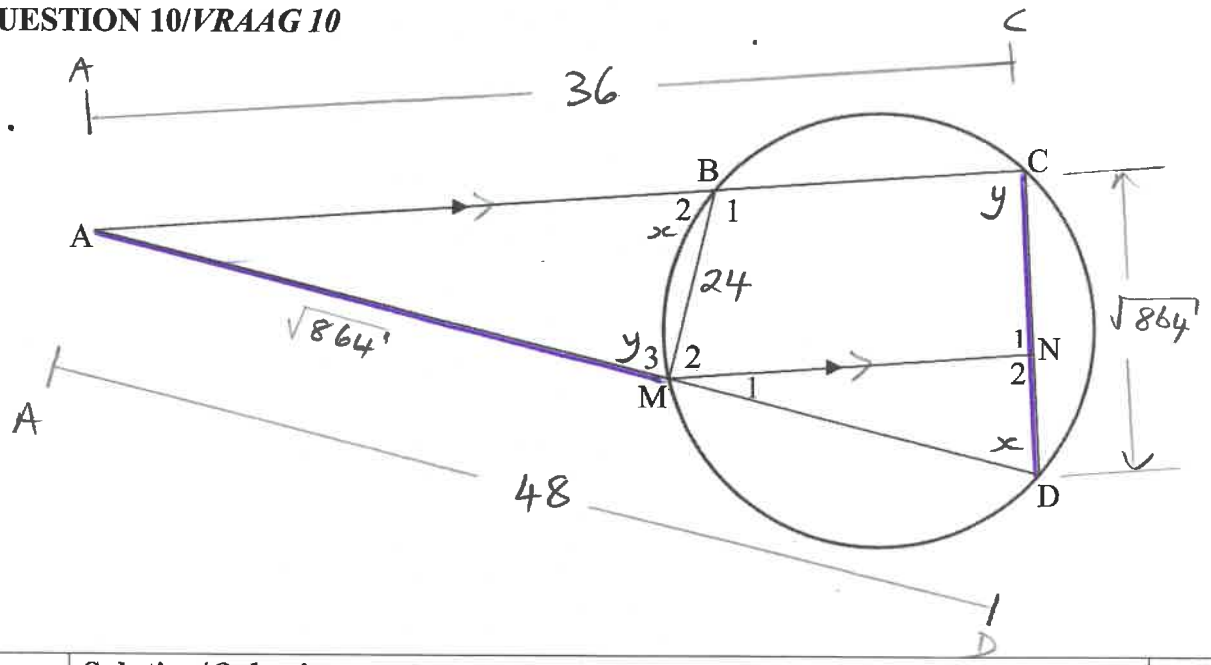
	Solution/*Oplossing*	Marks/ *Punte*
9.2.1	$\hat{T}_1 = y$ ✓S ✓R tan chord thm $\hat{T}_3 = y$ ✓S ✓R alt ^'s =, PR ∥ TS	(4)
9.2.2	$\hat{T}_2 = \hat{R}_1$ ✓S ✓R = chords = ^'s @ circum	(2)
9.2.3	$\hat{R}_1 = 180° - x - y$ ✓SR sum ^'s in Δ = 180° $\therefore \hat{T}_2 = 180° - x - y$ 9.2.2. $\hat{Q}_1 = x$ ✓SR ext ^ cyclic quad $x = 180° - x - y + y$ ext ^ Δ $2x = 180°$ $x = 90°$ ✓SR $\therefore \hat{S} = 90°$ $\therefore$ TR is diam ✓R conv ^ in semi ⊙ = 90°	(4)
		[15]

4

2

4

QUESTION 10/*VRAAG 10*



	Solution/*Oplossing*	Marks/ *Punte*
10.1	In Δ's $A\,B_2\,M_3$, $A\,D\,C$ 1. $\hat{A} = \hat{A}$ ✓S common 2 $\hat{B}_2 = \hat{D}$ ✓S ✓R ext $\angle$ cyclic quad 3. $\hat{M}_3 = \hat{C}$ ext $\angle$ cyclic quad $\therefore \Delta ABM \,|||\, \Delta ADC$ AAA ✓R	(4)
10.2	$\frac{AB}{AD} = \frac{BM}{DC} = \frac{MA}{CA}$ ✓S ✓S $\Delta ABM \,|||\, \Delta ADC$ $MA \cdot CD = BM \cdot AC$ $CD = MA$ ✓S given $CD \cdot CD = BM\ AC$ $CD^2 = BM \cdot AC$	(3)

4

3

NOTE on similar △ ratios :

If two triangles are similar, what can be said is :

$\frac{AB}{AD} = \frac{BM}{DC} = \frac{MA}{CA}$ $\quad \triangle ABM \,|||\, \triangle ADC$

What may NOT be said directly is :

× $\frac{AB}{BM} = \frac{AD}{DC}$ $\quad \triangle ABM \,|||\, \triangle ADC$

– there is no theorem which states that !

To get that $\frac{AB}{BM} = \frac{AD}{DC}$ you would have to go :

$\frac{AB}{AD} = \frac{BM}{DC} = \frac{MA}{CA}$ $\quad \triangle ABM \,|||\, \triangle ADC$

ie $\frac{AB}{AD} = \frac{BM}{DC}$

$\therefore \quad AB \cdot DC = BM \cdot AD$ cross multiply

$\therefore \quad \frac{AB}{BM} = \frac{AD}{DC}$

	Solution/*Oplossing*	**Marks/ *Punte***
10.3	$CD^2 = BM \cdot AC$ 10 2 $= 24 \cdot 36$ $CD = \sqrt{864}$ ✓ $\frac{CN}{CD} = \frac{AM}{AD}$ ✓S ✓R line ∥ 1 side of Δ $\frac{CN}{\sqrt{864}} = \frac{\sqrt{864}}{48}$ ✓sub $48 \cdot CN = 864$ ✓ $CN = 18$ ✓	6 (6)
		[13]

TOTAL/*TOTAAL*: 150